Addition of Operators :-

Addition of two operators gives a new operator defined as $(\hat{A}+\hat{B})f(x) = \hat{A}f(x) + \hat{B}f(x)$

Let operator $\hat{A}$ is $\frac{d}{dx}$ and $\hat{B}$ is 'x' respectively and function $f(x)$ is x^2.

then $\left(\frac{\partial}{\partial x}+x\right)(x)^2 = \frac{\partial x^2}{\partial x} + x^3$

$= 2x + x^3$

Substraction of Operators :-

The substraction of two operators gives a new operator defined as

$$(\hat{A}-\hat{B})f(x) = \hat{A}f(x) - \hat{B}f(x)$$

Let operator $\hat{A}$ is $\frac{d}{dx}$

and $\hat{B}$ is $\frac{d^2}{dx^2}$

& $f(x)$ is $\sin x$

then,

$$\left(\frac{d}{dx} - \frac{d^2}{dx^2}\right)\sin x = \frac{d\sin x}{dx} - \frac{d^2(\sin x)}{dx^2}$$

$$= \cos x - (-\sin x)$$

$$= \cos x + \sin x$$

—x—

Multiplication of Operators:-

Multiplication of two operators means operations by two operators, one after the another, the order of operation being from right to left.

For example,

$$\hat{A}\,\hat{B}\,f(x)$$

In the above case the function $f(x)$ is first operated by $\hat{B}$ to give a new function $g(x)$ then function $g(x)$ is operated by $\hat{A}$ to give $h(x)$ as the final function.

$$\hat{A}\,\hat{B}\,f(x) = \hat{A}\,[\hat{B}\,f(x)]$$
$$= \hat{A}\,[g(x)]$$
$$= h(x)$$

let $\hat{A}$ is $\frac{d}{dx}$

$\hat{B}$ is $\ln$

and $f(x)$ is x^2

then, $\hat{A}\,\hat{B}\,f(x) = \frac{d}{dx}\log(x^2)$

$$= \frac{d}{dx}[\log x^2]$$
$$= \frac{d}{dx}(2\log x)$$
$$= 2\frac{d}{dx}\log x$$
$$= 2 \times \frac{1}{x}$$
$$= \frac{2}{x}$$

Commutative Property of Operators.

Two operators are such that the result of their successive operations is the same irrespective of the order of their operations, then the two operators are said to be Commutative.

Example :- For the two operators $\hat{A}$ and $\hat{B}$ are said to be Commutative

$$\hat{A}\hat{B}f(x) = \hat{B}\hat{A}f(x)$$

Let $\hat{A}$ is $\frac{d}{dx}$ and $\hat{B}$ is $\frac{d^2}{dx^2}$, and $f(x) = \sin x$

Hence,
$$\hat{A}\hat{B}f(x) = \frac{d}{dx}\left[\frac{d^2}{dx^2}\sin x\right]$$
$$= \frac{d}{dx}\left[-\sin x\right] \quad \text{———} \quad (1)$$
$$= -\cos x$$

and,
$$\hat{B}\hat{A}f(x) = \frac{d^2}{dx^2}\left[\frac{d}{dx}\sin x\right]$$
$$= \frac{d^2}{dx^2}\cos x \quad \text{———} \quad (2)$$
$$= -\cos x$$

Here the result is same, i.e $(-\cos x)$ so operator $\hat{A}$ and $\hat{B}$ are said to commute.

The significance of the commutating operators lies in the fact that the observables corresponding to their eigen values can be determined precisely and simultaneously.

—x—